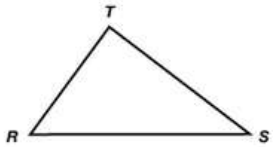
Objectives: To find the relationship between the measure of angles and sides opposite of them in a triangle, and to determine whether the given measures could form a triangle.

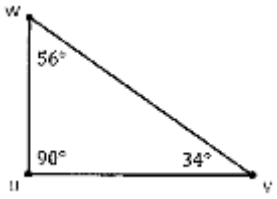
Opposite angles and sides

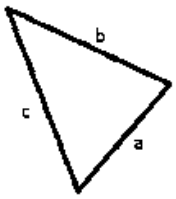
Recall that whenever we use the Pythagorean Theorem, we need to determine which side is the hypotenuse of the triangle so that we can plug in the values of the right triangle in the correct variable in the Pythagorean Theorem. In order to do so, we start from the corner of the right angle, and we just go straight from the corner of the right angle to the middle of the side across. That is the method that we would follow in order to determine which angle and sides in a triangle are opposite of each other.

Normally when we label sides and angles, we use capital letters for the angles, and lowercase letters for the sides. The opposite sides and angles tend to be the same letter of the alphabet.

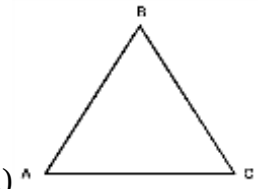
Example 1: Label the opposite sides or angles in every triangle given.

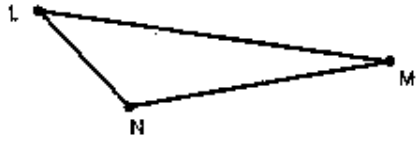
A) 

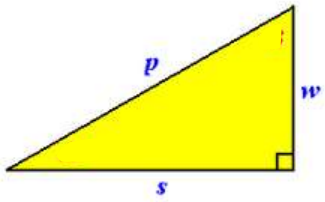
B) 

C) 

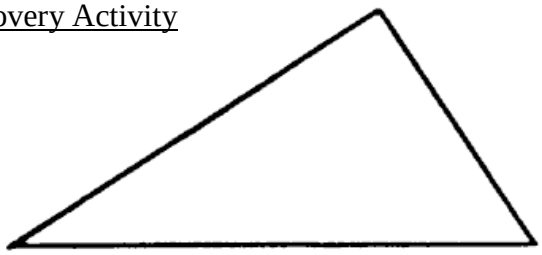
Practice 1: Label the opposite sides or angles in every triangle given.

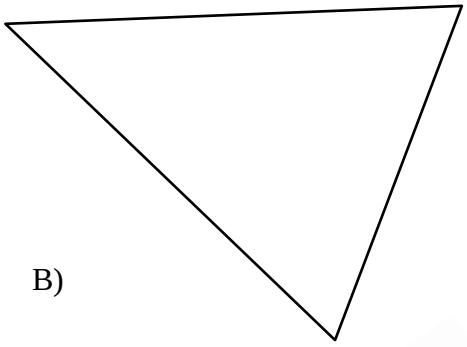
A) 

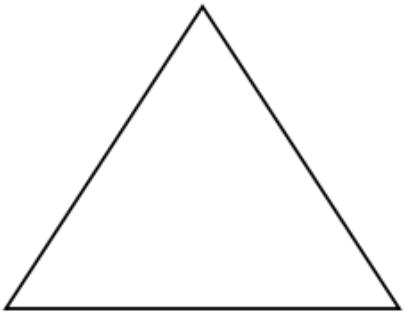
B) 

C) 

Discovery Activity

A) 

B) 

C) 

D) 

1) Measure every angle of the triangles above, and write their measures inside the angles.
2) Name the angles for each triangle as follows:
   the smallest angle=A
   the medium angle=B
   the largest angle=C
3) Label the sides of sides opposite of each angle based on the name of the angle opposite each side (i.e., the side across from angle A would be *a*, the side across from angle B is *b*, and the side across from angle C would be *c*).
4) Measure each side, and write the measures next to the name of each side.
5) List the measure of the sides of every triangle from smallest to large.

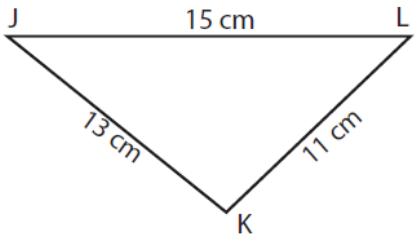
	Smallest Side	Medium Side	Largest Side
Triangle A			
Triangle B			
Triangle C			
Triangle D			

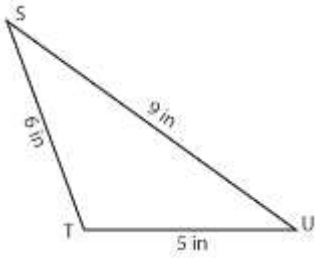
5) What do you notice about the measures of the angles and the measures of the sides opposite the angles?

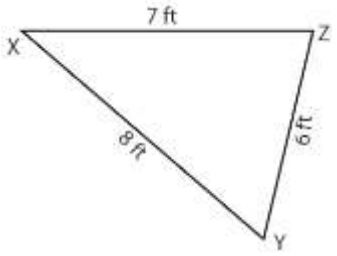
Angle and side relations

- The smallest angle in a triangle will be across from the _____ side of the triangle.
- The medium angle in a triangle will be across from the _____ side of the triangle.
- The largest angle in a triangle will be across from the _____ side of the angle.
- If at least two angles measure the same, the sides across from those angles will _____ _____; in other words, _____ angles are across from _____ sides.

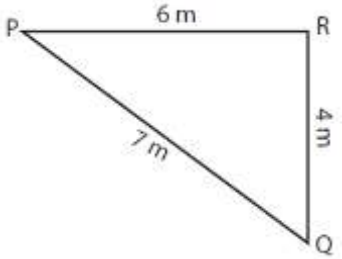
Example 2: Arrange the measures of the angles from smallest to largest.

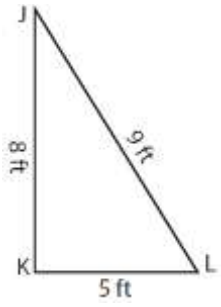
A) 

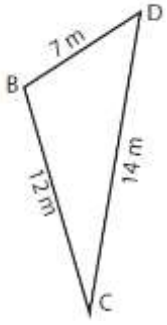
B) 

C) 

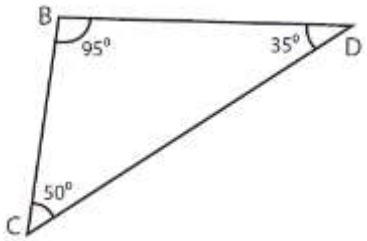
Practice 2: Arrange the measures of the angles from smallest to largest.

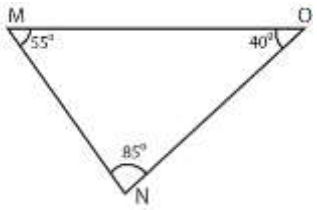
A) 

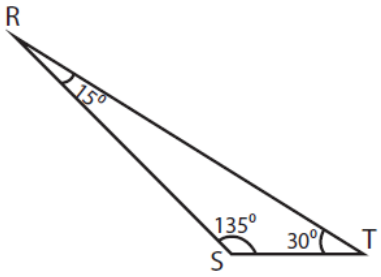
B) 

C) 

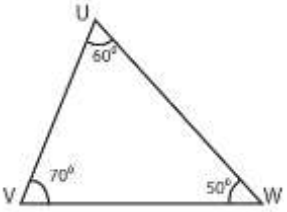
Example 3: Arrange the measures of the sides from smallest to largest.

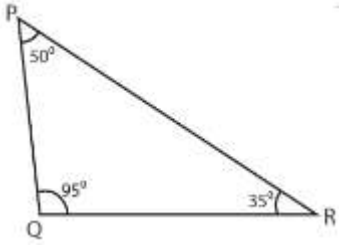
A) 

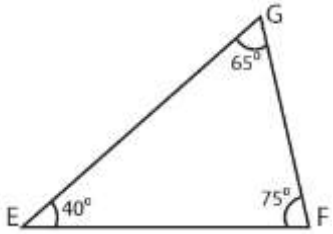
B) 

C) 

Practice 3: Arrange the measures of the sides from smallest to largest.

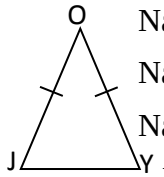
A) 

B) 

C) 

All of the triangles in examples 2 and 3 above were scalene triangles. The same relationship between measures of angles and their opposite sides applies to isosceles and equilateral triangles. The only difference is that in those types of triangles, we have at least one pair of angles and sides that measure the same.

Vocabulary Terms for an Isosceles Triangle

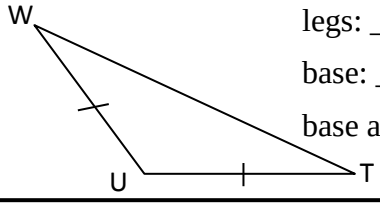


Name the legs: _____

Name the base: _____

Name the base angles: _____

Name the vertex angle:_____



legs: _____

base: _____

base angles: _____

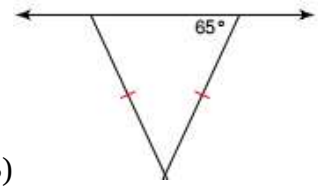
vertex angle:_____

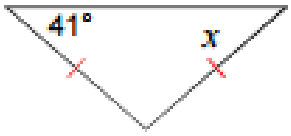
Isosceles Triangle Theorem

If two sides of a triangle are _____, then the angles opposite those sides are _____.

Example 4 and Practice 4: Find the measure of the missing base angle.

A) 

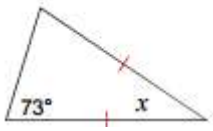
B) 

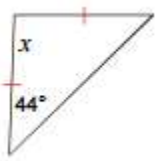
C) 

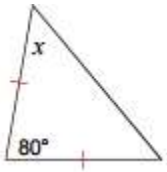
In an isosceles triangle, we only need one of the measures of the interior angles in order to find the measure of all three angles.

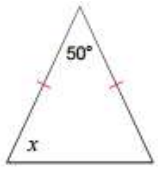
- If we have the measure of one of the base angles, then the other base angle would be congruent to that, and the vertex angle would be equal to the sum of the base angles subtracted from 180.
- If we know the measure of the vertex angle, we would have to subtract that measure from 180, and then divide by two – that would give us the measure of each one of the base angles.

Example 5: Find the measure of the missing angles.

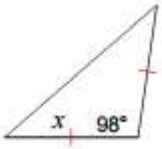
A) 

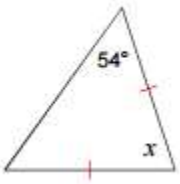
B) 

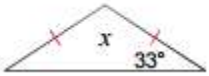
C) 

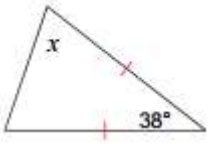
D) 

Practice 5: Find the measure of the missing angles.

A) 

B) 

C) 

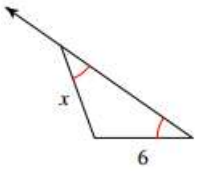
D) 

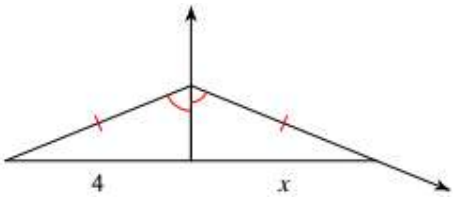
Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example and Practice 6: Find the value of the side represented by the variable.

A) 

B) 

C) 

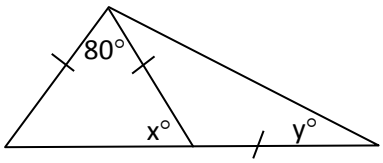
Equilateral Triangles

Recall the equilateral triangles are triangles with all sides congruent. Recall also that isosceles triangles are triangles with *at least* two sides congruent, which means that equilateral triangles are isosceles triangles; consequently, equilateral triangles have the properties of isosceles triangles; therefore, the angles opposites of congruent sides are congruent, and viceversa. In equilateral triangles, however, all the sides are congruent and all the angles are congruent; as a result:

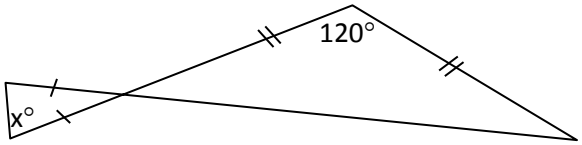
Corollary 4.3 – A triangle is equilateral if and only if it is equiangular.

Corollary 4.4 – Each angle of an equilateral triangle measures 60°.

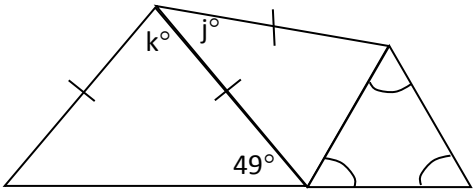
Practice 7: Solve for x and y.



Practice 8: Solve for x.



Practice 9: Solve for j and k.



Practice 10: Solve for *w*, *x* and *y* if ΔWHT is equilateral.